III. *On the law of the partial polarization of light by reflexion.* By DAVID BREWSTER, *LL.D. F.R.S. L. & E.*

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IN the year 1815 I communicated to the Royal Society a series of experiments on the polarization of light by successive reflexions, which contain the germ of the investigations, the results of which I now propose to explain.

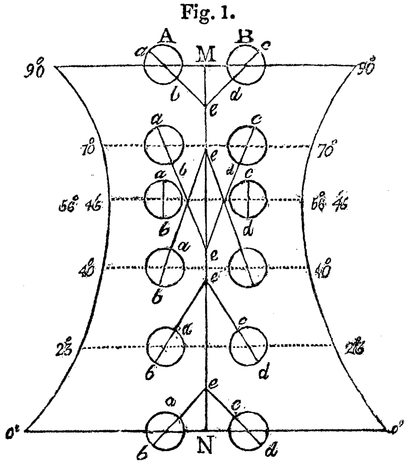
From these experiments it appeared that a given pencil of light could be wholly polarized at any angle of incidence, provided it underwent a sufficient number of reflexions, either at angles wholly above or wholly below the maximum polarizing angle, or at angles partly above and partly below that angle; and it was scarcely possible to resist the conclusion, that the light not polarized by the first reflexion had suffered a physical change at each action of the reflecting force which brought it nearer and nearer to the state of complete polarization. This opinion, however, which I have always regarded as demonstrable, appeared in a different light to others. Guided probably by an experimental result, apparently though not really hostile to it, Dr. YOUNG and MM. BIOT, ARAGO, and FRESNEL, have adhered to the original opinion of MALUS, that the reflected and refracted pencils consist partly of light wholly polarized, and partly of light in its natural state; and more recently Mr. HERSCHEL has given the weight of his opinion to the same view of the subject.

Under these circumstances I have often returned to the investigation with renewed zeal; but though the frequent repetition of my experiments has more and more convinced me of the truth of the conclusions which I drew from them, yet I have not till lately been able to place the subject in a satisfactory aspect, and to connect it with general laws, which give a mathematical form to this fundamental branch of the science of polarization.

If we consider a pencil of natural light as divided into two pencils polarized in rectangular planes by the action of a doubly refracting crystal, and conceive the light of these two pencils to return back through the crystal, it will obviously emerge in the state of natural light. When we examine the pencil thus recomposed, or when we examine a pencil consisting of two oppositely polarized pencils superposed, we shall find that they comport themselves under every analysis exactly like common light; so that we are entitled to assume such a pencil as the representative of natural light, and to consider every thing that can be established respecting the one, as true respecting the other.

In applying this principle to the analysis of the phænomena produced by reflexion, I placed the planes of polarization of the compound beam in the plane of reflexion; but though this led to some interesting conclusions, it did not develope any general law. I then conceived the idea of making the plane of reflexion bisect the right angle formed by the planes of polarization; and in this way I observed a series of symmetrical effects at different angles of incidence, which threw a broad light over the whole subject.

Fig. 1.



In order to explain these results, let AB (Fig. 1.) represent the two pencils of oppositely polarized light as separated by double refraction; let *a b*, *c d* be the directions of their planes of polarization, forming a right angle *a e c*, and let the plane of reflexion MN, of a surface of plate glass, bisect the angle *a e c*, so that the planes *a b*, *c d* form angles of + 45° and − 45° with the plane MN. Let a rhomb of calcareous spar have its principal section now placed in the plane of reflexion.

At an incidence of 90°, reckoned from the perpendicular, the reflected images of A and B suffer no change, the angle *a e c* is still a right angle, and the four pencils formed by the calcareous spar are all of equal intensity. As the incidence however diminishes, the angle *a e c* diminishes also, and the ordinary and extraordinary images of A and B differ in intensity. At an incidence of 80° for example, the angle *a e c* is reduced from 90° to 66°; at 70° it has been reduced to 40°, and at 56° 45′, the maximum polarizing angle, it has been reduced to 0°; that is, the planes of polarization *a b*, *c d* are now parallel. Below the polarizing angle, at 50°, the axes are again inclined to each other, and form an angle of 22°. At 40° they form an angle of 50°, and at 0°, or a perpendicular incidence, they are again brought back to their primitive inclination of 90°. Taking M N to represent the quadrant of incidence from 90° at M, to 0° at N, the curves, 90°, 0°, show the progressive change which takes place in the planes of polarization, the plane of polarization being a tangent to the curve at the incidence which corresponds to any particular point of it.

When we employ a surface of diamond in place of glass, the inclination of the axes *a b*, *c d* is reduced to 46° at an incidence of 80°, to 8° at an incidence of 70°, and at 67° 43′ the axes become parallel.

Such being the action of the reflecting forces upon A and B taken separately, let us now consider them as superposed and forming natural light. At 90° and 0° of incidence, the reflecting force produces no change in the inclination of their axes or planes of polarization; but at 56° 45′ in the case of glass, and 67° 43′ in the case of diamond, the axes of all the particles are brought into a state of parallelism with the plane of reflexion; and consequently when the image which they form is viewed by the rhomb of calcareous spar, they will all pass into the ordinary image, and thus prove that they are wholly polarized in the plane of reflexion.

All this is entirely conformable to what has been long known: but we now see that the total polarization of the reflected pencil at an angle whose tangent is the index of refraction, is effected by turning round the planes of polarization of one half of the light from right to left, and of the other half from left to right, each through an angle of 45°. Let us now see what takes place at those angles where the pencil is only partially polarized. At 80° for example, the angle of the planes *a b*, *c d* is 66°, that is, each plane of polarization has been turned round in opposite directions from an inclination of 45° to one of 33° with the plane of reflexion. The light has therefore suffered a physical change of a very marked kind, constituting now neither natural nor polarized light. It is not natural light, because its planes of polarization are not rectangular; it is not polarized light, because they are not parallel. It is a pencil of light having the physical character of one half of its rays being polarized at an angle of 66° to the other half. It will now be asked, how a pencil thus characterized can exhibit the properties of a partially polarized pencil, that is, of a pencil part of whose light is polarized in the plane of reflexion, while the rest retains its condition of natural light. This will be understood by replacing the analysing rhomb with its principal section in the plane of reflexion, and viewing through it the images A and B at 80° of incidence. As the axis of A is inclined 33° to MN or the section of the rhomb, the ordinary image of it will be much brighter than the extraordinary image, the intensity of each being in the ratio of $\cos^2 \varphi$ to $\sin^2 \varphi$, φ being the angle of inclination, or 33° in the present case. In like manner the ordinary image of B will be in the same ratio brighter than its extraordinary image, that is, by considering A and B in a state of superposition, the extraordinary image of a pencil of light reflected at 80° will be fainter than the ordinary image in the ratio of $\sin^2 33°$ to $\cos^2 33°$. But this inequality in the intensity of the two pencils is precisely what would be produced by a compound pencil, part of which is polarized in the plane of reflexion, and part of which is common light. When MALUS, therefore, and his successors analysed the pencil reflected at 80°, they could not do otherwise than conclude that it was partially polarized, consisting partly of light polarized in the plane of reflexion, and partly of natural light. The action of successive reflexions, however, afforded a more precise means of analysis, in so far as it proved that the portion of what was deemed natural light had in reality suffered a physical change, which approximated it to the state of polarized light; and we now see that the portion of what was called polarized light was only what may be called apparently polarized; for though it disappears, like polarized light, from the extraordinary image of the analysing prism, yet there is not a single particle of it polarized in the plane of reflexion.

These results must be admitted to possess considerable interest in themselves; but, as we shall proceed to show, they lead to conclusions of general importance. The quantity of light which disappears from the extraordinary image, is obviously the quantity of light which is really or apparently polarized at the given angle of incidence; and if we admit the truth of the law of repartition discovered by MALUS, and represented by $P_{oo} = P_o \cos^2 \varphi$, and $P_{oe} = P_o \sin^2 \varphi$, and if we can determine φ for substances of every refractive power, and for all angles of incidence, we may consider as established the mathematical law which determines the intensity of the polarized pencil, whatever be the nature of the body which reflects it,—whatever be the angle at which it is incident,—whatever be the number of reflexions which it suffers, and whether these reflexions are all made from one substance, or partly from one substance and partly from another.

The first step in this investigation is to determine the law according to which a reflecting surface changes the plane of polarization of a polarized ray. This subject was first examined by MALUS, but not with that success which attended most of his labours. Before I was acquainted with what had been done by M. FRESNEL, or with the experiments of M. ARAGO on glass and water, I had made a number of very careful experiments on the same subject, and had represented them by formulæ founded on the law of the tangents. These formulæ, however, I found to be defective; and I am persuaded, from a very extensive series of experiments, that the formulæ of FRESNEL are accurate expressions of the phenomena under every variation of incidence and refractive power. If i is the angle of incidence, i' the angle of refraction, x the primitive inclination of the plane of the polarized ray to the plane of reflexion, and φ the inclination to which that plane is brought by reflexion, then, according to FRESNEL, we have

$$\text{Tan}\ \varphi = \tan x \frac{\cos(i + i')}{\cos(i - i')}$$

When x is 45°, as in the preceding observations, then $\tan x = 1$, and we have

$$\text{Tan}\ \varphi = \frac{\cos(i + i')}{\cos(i - i')}.$$

In these formulæ, which are founded on the law of the tangents, $i + i'$ is the supplement of the angle which the reflected ray forms with the refracted ray; while $i - i'$ is the angle which the incident ray forms with the refracted ray, or the deviation produced by refraction.

These formulæ have been verified by M. ARAGO at ten angles of incidence upon Glass, and four upon Water; but his experiments were made only in the case where x is 45°, and where $\tan x$ disappears from the formula. As my experiments embrace a wider range of substances, and also the general case where x varies from 0° to 90°, I consider them as a necessary basis for a law of such extensive application.

The first series of experiments which I made was upon Plate Glass, in which the maximum polarizing angle was nearly 56°: hence I assume the index of refraction to be 1.4826. The following were the results:

Plate Glass.

Angle of Incidence.	Angle of Refraction.	Inclination of Plane of Polarization to Plane of Reflexion. Observed.	Computed.	Difference.
90°	0° 0′	45° 0′	45° 0′	0° 0′
88	42 23	43 4	42 49	+0 35
86	42 17	40 43	40 36	+0 7
84	42 8	38 47	38 22	+0 25
80	41 37	33 13	33 46	−0 33
75	40 40	28 45	27 41	+1 4
70	39 20	22 6	21 3	+1 3
65	37 41	14 40	13 53	+0 47
60	35 45	6 10	6 16	−0 6
56	34 0	0 0	0 0	0 0
50	31 22	9 0	9 0	0 0
45	28 29	16 55	16 31	+0 24
40	25 42	22 37	23 1	−0 24
30	19 43	32 25	33 19	−0 54
20	13 20	39 0	40 4	−1 4
10	6 44	44 0	43 49	+0 11

These results, obtained in every part of the quadrant, completely establish the accuracy of the formula. The differences are all within the limits of the errors of observation, and amount, at an average, to $32\frac{1}{2}'$ on each observation.

It is a curious circumstance, which I believe has not before been remarked, that at an incidence of 45° the deviation produced by refraction, or $i-i'$, is, in every substance, the complement of the angle of refraction i to 45°; and in the action of all substances upon polarized light at an incidence of 45°, the rotation of the plane of polarization of a pencil polarized + 45°, or − 45°, is equal to the angle of refraction; while the inclination of the plane of polarization to the plane of reflexion, or φ, is equal to the deviation $i-i$.

In order to establish the accuracy of the formula for different degrees of refractive power, I made the following experiments on Diamond, in which the index of refraction was 2.440.

Diamond.

Angle of Incidence.	Angle of Refraction.	Inclination of Plane of Polarization to Plane of Reflexion. Observed.	Calculated.	Difference.
90° 0′	24° 12′	45° 0′	45° 0′	0° 0′
85 0	24 6	34 30	33 56	+0 34
80 0	23 48	24 0	23 12	+0 48
75 0	23 19	14 30	13 8	+1 22
70 0	22 39	4 30	3 54	+0 36
67 43	22 17	0 0	0 0	0 0
60 0	20 47	12 30	11 41	+0 49
50 0	18 18	24 0	23 30	+0 30

These differences, which at an average amount to $46\frac{1}{2}'$, are also within the limits of the errors of observation.

In all these experiments the value of x was 45°; but in order to determine the law of variation for φ, when x varies from 0° to 90°, I took a crystal of quartz with a fine natural surface parallel to its axis; and I found that at an angle of incidence of 75°, and when x was 45°, the inclination of the plane of polarization to the plane of reflexion was 26° 20′. I then varied x, and obtained the following results:

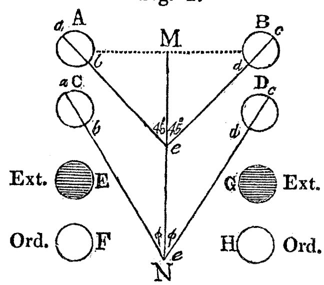
Values of x.	Inclination of Plane of Polarization. φ Observed.	φ Calculated.	Difference.
0°	0° 0′	0° 0′	0° 0′
10	4 54	4 29	+0 25
20	10 0	10 16	−0 16
30	15 50	16 2	−0 12
35	20 0	19 12	+0 48
40	23 30	22 40	+0 50
45	26 20	26 27	−0 7
50	30 0	30 40	−0 40
55	35 30	35 23	+0 7
60	40 0	40 45	−0 45
70	53 0	53 49	−0 49
80	70 0	70 29	−0 29
90	90 0	90 0	0 0

In these experiments the average error does not exceed half a degree. The third column is computed by the formula tan φ = (tan 26° 27′) tan x.

From these experiments it appears that the formula expresses with great accuracy all the changes in the planes of polarization which are produced by a single reflexion, and we may therefore apply it in our future investigations.

Let us now suppose that a beam of common light composed of two portions A, B, (Fig. 2.) polarized + 45° and − 45° to the plane of reflexion, is incident on a plate of glass at such an angle that the reflected pencil composed of C and D has its planes of polarization inclined at an angle φ to the plane M N. When a rhomb of calcareous spar has its principal section in the plane M N, it will divide the image C into an extraordinary pencil E and an ordinary one F; and the same will take place with D, G being its extraordinary and H its ordinary image. If we represent the whole of the reflected pencil or C + D by 1, then C = $\frac{1}{2}$, D = $\frac{1}{2}$, E + F = 1, and G + H = 1. But since the planes of polarization of C and D are each inclined φ degrees to the principal section of the rhomb, the intensity of the light of the doubly refracted pencils will be as $\sin^2\varphi : \cos^2\varphi$; that is, the intensity of E will be $\frac{1}{2}\sin^2\varphi$, and that of F, $\frac{1}{2}\cos^2\varphi$. Hence it follows that the difference of these pencils, or $\frac{1}{2}\sin^2\varphi - \frac{1}{2}\cos^2\varphi$, will express the quantity of light which has passed from the extraordinary image E into the ordinary one F, that is, the quantity of light apparently polarized in the plane of reflexion M N. But as the same is true of the pencil D, we have $2\left(\frac{1}{2}\sin^2\varphi - \frac{1}{2}\cos^2\varphi\right)$ or $\sin^2\varphi - \cos^2\varphi$ for the whole of the polarized light in a pencil of common light C + D. Hence, since $\sin^2\varphi + \cos^2\varphi = 1$ and $\cos^2\varphi = 1 - \sin^2\varphi$, we have for the whole quantity of polarized light

Fig. 2.



$$Q = 1 - 2\sin^2\varphi.$$

But
$$\operatorname{Tan}\varphi = \tan x\,\frac{\cos(i+i')}{\cos(i-i')}$$

And as
$$\operatorname{Tan}^2\varphi = \frac{\sin^2\varphi}{\cos^2\varphi},\ \text{and}\ \sin^2\varphi + \cos^2\varphi = 1,$$

we have the quotient and the sum of the quantities $\sin^2 \phi$ and $\cos^2 \phi$, by which we obtain

$$\text{Sin}^2 \phi = \frac{1}{\dfrac{1}{\left(\tan x \dfrac{\cos(i+i')}{\cos(i-i')}\right)^2} + 1} = \frac{\left(\tan x \dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}{1 + \left(\tan x \dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}$$

That is $$Q = 1 - 2 \frac{\left(\tan x \dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}{1 + \left(\tan x \dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}$$

As the quantity of reflected light is here supposed to be 1, we may obtain an expression of Q in terms of the incident light by adopting the formula of Fresnel for the intensity of a reflected ray. Thus

$$Q = \tfrac{1}{2}\left(\frac{\sin^2(i-i')}{\sin^2(i+i')} + \frac{\tan^2(i-i')}{\tan^2(i+i')}\right)\left(1 - 2 \frac{\left(\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}{1 + \left(\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}\right)$$

As $\tan x = 1$ in common light, it is omitted in the preceding formula.

This formula may be adapted to partially polarized rays, that is, to light reflected at any angle different from the angle of maximum polarization, provided we can obtain an expression for the quantity of reflected light.

M. Fresnel's general formula has been adapted to this species of rays, by considering them as consisting of a quantity a of light completely polarized in a plane making the angle x with that of incidence, and of another quantity $1 - a$ in the state of natural light. Upon this principle it becomes

$$I = \frac{\sin^2(i-i')}{\sin^2(i+i')} \cdot \frac{1 + a\cos^2 x}{2} + \frac{\tan^2(i-i')}{\tan^2(i-i')} \cdot \frac{1 - a\cos^2 x}{2}.$$

But as we have proved that partially polarized rays are rays whose planes of polarization form an angle of $2x$ with one another as already explained, x being greater or less than 45°, we obtain a simpler expression for the intensity of the reflected pencil, viz. the very same as that for polarized light.

$$I = \frac{\sin^2(i-i')}{\sin^2(i+i')} \cos^2 x + \frac{\tan^2(i-i')}{\tan^2(i+i')} \sin^2 x$$

Hence we have

$$Q = \left(\frac{\sin^2(i-i')}{\sin^2(i+i')}\cos^2 x + \frac{\tan^2(i-i')}{\tan^2(i+i')}\sin^2 x\right)\left(1 - 2\,\frac{\left(\tan x\,\frac{\cos(i+i')}{\cos(i-i')}\right)^2}{1+\left(\tan x\,\frac{\cos(i+i')}{\cos(i-i')}\right)^2}\right)$$

This formula is equally applicable to a single pencil of polarized light of the same intensity as the pencil of partially polarized light. In all these cases it expresses the quantity of light really or apparently polarized in the plane of reflexion.

In order to show the quantity of light polarized at different angles of incidence, I have computed the following table for common light, and suited to glass in which $m = 1.525$.

PLATE GLASS.

Angle of Incidence i.	Angle of Refraction i'.	Inclination of Plane of Polarization to Plane of Reflexion, φ.	Quantity of Light reflected out of 1000 Rays.	Quantity of Polarized Light Q.	Ratio of Polarized to Reflected Light.
0° 0′	0° 0′	45° 0′	43.23	0	0
10 0	6 32	43 51	43.39	1.74	0.04000
20 0	12 58	40 13	43.41	7.22	0.16618
25 0	16 5	37 21	43.64	11.6	0.26388
30 0	19 8½	33 40	44.78	17.25	0.3853
35 0	22 6	29 8	46.33	24.37	0.5260
40 0	24 56	23 41	49.10	33.25	0.6773
45 0	27 37½	17 22½	53.66	44.09	0.82167
50 0	30 9	10 18	61.36	57.36	0.9360
56 45	33 15	0 0	79.5	79.5	1.000
60 0	34 36	5 4½	93.31	91.6	0.9628
65 0	36 28	12 45	124.86	112.7	0.90258
70 0	38 2	18 32	162.67	129.80	0.79794
75 0	39 18	26 52	257.26	152.34	0.59154
78 0	39 54	30 44	329.95	157.67	0.47786
79 0	40 4	31 59	359.27	157.69	0.43892
80 0	40 13	33 13	391.7	156.6	0.40000
82 44	40 35	36 22	499.44	145.4	0.29112
84 0	40 42	38 2	560.32	134.93	0.2408
85 0	40 47	39 12	616.28	123.75	0.2008
86 0	40 51	40 22.7	676.26	108.67	0.16068
87 0	40 54	41 32	744.11	89.83	0.12072
88 0	40 57½	42 42	819.9	65.9	0.0804
89 0	40 58	43 51	904.81	36.32	0.04014
90 0	40 58	45 0	1000.0	0	0.0000

As the preceding formula is deduced from principles which have been either established by experiment or confirmed by it, it may be expected to harmonize with the results of observation. At all the limits where the pencil is either wholly polarized or not polarized at all, it of course corresponds with experiment: but though in so far as I know there have been no absolute measures taken of the quantity of polarized light at different incidences, yet we are fortunately in possession of a set of experiments by M. Arago, who has ascertained the angles above and below the polarizing angle at which glass and water polarize the same proportion of light. In no case has he measured the absolute quantity of the polarized rays; but the comparison of the values of Q at those angles at which he found them in equal proportions, will afford a test of the accuracy of the formula. This comparison is shown in the following table, in which col. 1. contains the angles at which the reflecting surface polarizes equal proportions of light; col. 2. the values of ϕ or the inclination of the planes of polarization; and col. 3. the intensities of the polarized light computed from the formula.

	Angles of Incidence i.	Inclination of Planes of Polarization to M N, or ϕ.	Proportion of Polarized Light or Q.
Glass: No. 1.	82° 48′	37° 33′	.2572
	24 18	37 21	.2637
No. 2.	82 5	36 47	.2828
	26 6	36 0	.3090
No. 3.	78 20	32 38	.4186
	29 42	33 1	.4064
Water: No. 4.	86 31	41 54	.1080
	16 12	41 27	.1236

The agreement of the formula with experiments made with as great accuracy as the subject will admit must be allowed to be very satisfactory. The differences are within the limits of the errors of observation, as appears from the following table:

	Deviations from Experiment.	Part of the whole Light.
Glass: No. 1.	0.0065	$\frac{1}{154}$
No. 2.	0.0262	$\frac{1}{38}$
No. 3.	0.0122	$\frac{1}{82}$
Water: No. 4.	0.0156	$\frac{1}{64}$

M. Arago has concluded, from the experiments above stated, that equal proportions of light are polarized at equal angular distances from the angle of complete polarization. Thus in Glass No. 1. the mean of 82° 48′ and 24° 18′ is 53° 33′, which does not differ widely from the maximum polarizing angle, or 55°, which M. ARAGO considers as the maximum polarizing angle of the glass *. In order to compare this principle with the formula, I found that in Water No. 4. the angle which polarizes almost exactly the same proportion of light as the angle of 86° 31′, is 15° 10′, the value of φ being 41° 54′ at both these angles; but the mean of these is 50° 50′ in place of 53° 11′; so that the rule of M. ARAGO cannot be regarded as correct, and cannot therefore be employed, as he proposes, to determine the angle of complete polarization †.

The application of the law of intensity to the phenomena of the polarization of light by successive reflexions, forms a most interesting subject of research. No person, so far as I know, has made a single experiment upon this point, and those which I have recorded in the Philosophical Transactions for 1815, have, I believe, never been repeated. All my fellow labourers, indeed, have overlooked them as insignificant, and have even pronounced the results which flow from them to be chimerical and unfounded. Those immutable truths, however, which rest on experiment, must ultimately have their triumph; and it is with no slight satisfaction, that, after fifteen years of unremitted labour, I am enabled not only to demonstrate the correctness of my former experiments, but to present them as the necessary and calculable results of a general law.

When a pencil of common light has been reflected from a transparent surface, at an angle of 61° 3′ for example, it has experienced such a physical change, that its planes of polarization form an angle of 6° 45′ each with the plane of reflexion. When it is incident on another similar surface at the same angle, it is no longer common light in which $x = 45°$, but it is partially polarized light in which $x = 6° 45'$. In computing therefore the effect of the second reflexion, we must take the general formula $\tan\varphi = \tan x \left(\frac{\cos(i+i')}{\cos(i-i')}\right)$; but, as the value of x is always in the same ratio to the value of φ, however great be the number of reflexions, we have $\tan\theta = \tan^n\varphi$ for the inclination θ to the plane of reflexion produced by any number of reflexions n,

* Hence we have assumed $m = 1.428$, the tangent of 55°, in the preceding calculations.

† It is obvious that the rule can only be true when $m = 1.000$; so that its error increases with the refractive power.

φ being the inclination for one reflexion. Hence when θ is given by observation, we have $\tan \varphi = \sqrt[n]{\theta}$. The formula for any number n of reflexions is therefore $\tan \theta = \left(\frac{\cos (i + i')}{\cos (i + i)}\right)^n$. It is evident that θ never can become equal to 0°; that is, that the pencil cannot be so completely polarized by any number of reflexions at angles different from the polarizing angle, as it is by a single reflexion at the polarizing angle; but we shall see that the polarization is sensibly complete in consequence of the near approximation of θ to 0°.

I found, for example, that light was polarized by two reflexions from glass at an angle of 61° 3′, and 60° 28′ by another observation. Now in these cases we have

	θ after 1st Reflexion.	θ after 2nd Reflexion.	Quantity of Unpolarized Light.
Two reflexions at 61° 3′	6° 45′	0° 47′	0.00037
60 28	5 38	0 33	0.00018

The quantity of unpolarized light is here so small as to be quite inappreciable with ordinary lights.

In like manner I found that light was completely polarized by five reflexions at 70°. Hence by the formula we have

	Values of θ.	Unpolarized Light.
1 reflexion at 70°	20° 0′	0.23392
2	7 32	0.03432
3	2 45	0.00460
4	1 0	0.00060
5	0 22	0.00008

The quantity of unpolarized light is here also unappreciable after the 5th reflexion.

In another experiment I found that light was wholly polarized by the separating surface of glass and water at the following angles:

	Values of θ.	Unpolarized Light.
By 2 reflexions at 44° 51′	0° 56′	0.0005
By 3 . . . . . 42 27	0 26	0.0001

In all these cases the successive reflexions were made at the same angle; but the formula is equally applicable to reflexions at different angles,—

1. When both the angles are greater than the polarizing angle.

	θ	Unpolarized Light.
1 reflexion at 58° 2′, and 1 at 67° 2′	0° 34′	0.0002

2. When one of the angles is above and the other below the polarizing angle.

	θ	Unpolarized Light.
1 reflexion at 53°, and 1 at 58° 2′ . .	0° 12′ . .	0.000024

This experiment requires a very intense light, for I find in my journal that the light of a candle is polarized at 53° and 78°.

In reflexions at different angles, the formula becomes $\tan \theta = \frac{\cos (i + i')}{\cos (i - i')} \times \frac{\cos (I + I')}{\cos (I + I')}$, I and i being the angles of incidence. In like manner if a, b, c, d, e, &c. are the values of φ or θ for each reflexion, or rather for each angle of incidence, we shall have the final angle or $\tan \theta = \tan a \times \tan b \times \tan c \times \tan d$, &c.

It is scarcely necessary to inform the reader that when a pencil of light reflected at 58° 2′ is said to be polarized by another reflexion at 67° 2′, it only means, that this is the angle at which complete polarization takes place in diminishing the angle gradually from 90° to 67° 2′, and that even this angle of 67° 2′ will vary with the intensity of the original pencil, with the opening of the pupil, and with the sensibility of the retina. But when it shall be determined experimentally at what value of φ, or rather at what value of Q, the light entirely disappears from the extraordinary image, we shall be able by inverting the formula to ascertain the exact number of reflexions by which a given pencil of light shall be wholly polarized.

As the value of Q depends on the relation of i and i', that is, on the index of refraction, and as this index varies for the different colours of the spectrum, it is obvious that Q will have different values for these different colours. The consequence of this must be, that in bodies of high dispersive powers, the unpolarized light which remains in the extraordinary image, and also the light which forms the ordinary image, must be coloured at all incidences; the colours being most distinct near the maximum polarizing angle. This necessary result of the formula, I found to be experimentally true in oil of cassia, and various highly dispersive bodies. In realgar for example φ is $= 0$ at an angle of 69° 0′ for blue light, at 68° 37′ for green light, and at 66° 49′ for red light. Hence there can be no angle of complete polarization for white light, which I also found to be the case by experiment; and as Q must at different angles of incidence have different values for the different rays, the unpolarized light must be composed of a certain portion of each different colour, which may be easily determined by the formula.

Such are the laws which regulate the polarization of light by reflexion from the first surfaces of bodies that are not metallic. The very same laws are applicable to their second surfaces, provided that the incident light has not suffered previous or subsequent refraction from the first surface. The sine of the angle at which ϕ or Q has a certain value by reflexion from the second surface, is to the sine of the angle at which they have the same value at the first surface, as unity is to the index of refraction. Hence ϕ and Q may be determined by the preceding formulæ after any number of reflexions, even if some of the reflexions are made from the first surface of one body and the second surface of another.

When the second surface is that of a plate with parallel or inclined faces, its action upon light presents curious phenomena, the law of which I have determined. I refer of course to the action of the second surface at angles less than that which produces total reflexion. This action has hitherto remained uninvestigated. It has been hastily inferred, however, from imperfect data; and the erroneous inference forms the basis of some optical laws, which are considered to be fully established.

Among the various results of the preceding investigation, there is one which seems to possess some theoretical importance. If we consider polarized rays as those whose planes of polarization are parallel, then it follows that light cannot be brought into such a state by any number of reflexions, or at any angle of incidence, excepting at the angle of complete polarization. At all other angles the light which seems to be polarized, by disappearing from the extraordinary image of the analysing rhomb, is distinguished from really polarized light, by the property of its planes of polarization forming an angle with each other and with the plane of reflexion. At the polarizing angle, for example, of 56° 45′ in glass, the light reflected is 79.5 rays, and it is completely polarized, because the planes of polarization of all the rays are parallel; but at an angle of incidence of 80°, where 392 rays are reflected, no fewer than 157 appear to be polarized, though their planes of polarization are inclined 66° 26′ to each other, or 33° 13′ to the plane of reflexion. This appearance of polarization, when the rays have only suffered a displacement in their planes of polarization from an angle of 90°, which approximates them to the state of polarized light, arises from the law which regulates the repartition of polarized light between the ordinary and extraordinary images produced by double refraction, and shows that the analysing crystal is not sufficient to distinguish light completely polarized from light in a state of approach to polarization. The difference, however, between these two kinds of light is marked by most distinctive characters, and will be found to show itself in some of the more complex phenomena of interference.

In my paper of 1815, already referred to, I was led by a distant view of the phenomena which I have now developed, to consider common light as composed of rays in every state of positive and negative polarization*; and upon this principle the whole of the phenomena described in this paper may be calculated with the same exactness as upon the supposition of two oppositely polarized pencils. Nothing indeed can be simpler than such a principle. The particles of light have planes, which are acted upon by the attractive and repulsive forces residing in solid bodies; and as these planes must have every possible inclination to a plane passing through the direction of their motion, one half of them will be inclined — to this plane, and the other half +. When light in such a state falls upon a reflecting surface, the — and the + particles have each their planes of polarization brought more or less into a state of parallelism with the plane of reflexion, in consequence of the action of the repulsive force upon one side or pole of the particle through which the plane passes; while in the particles which suffer refraction, the same sides or poles are by the action of the attractive force drawn downwards, so as to increase the inclination of their planes relative to the plane of incidence, and bring them more or less into a state of parallelism with a plane perpendicular to that of refraction.

The formulæ already given, and those for refracted light which are contained in another paper, represent the laws according to which the repulsive and attractive forces change the position of the planes of polarization; and as we have proved that the polarization is the necessary consequence of these planes being brought into certain positions, we may regard all the various phenomena of the polarization of light by reflexion and refraction, as brought under the dominion of laws as well determined as those which regulate the motions of the planets.

Allerly, December 25, 1829.

* M. Biot has followed me in this opinion. See *Traité de Physique*, tom. iv. p. 304.